3- Models of Quantum Computation

Models of Quantum Computation

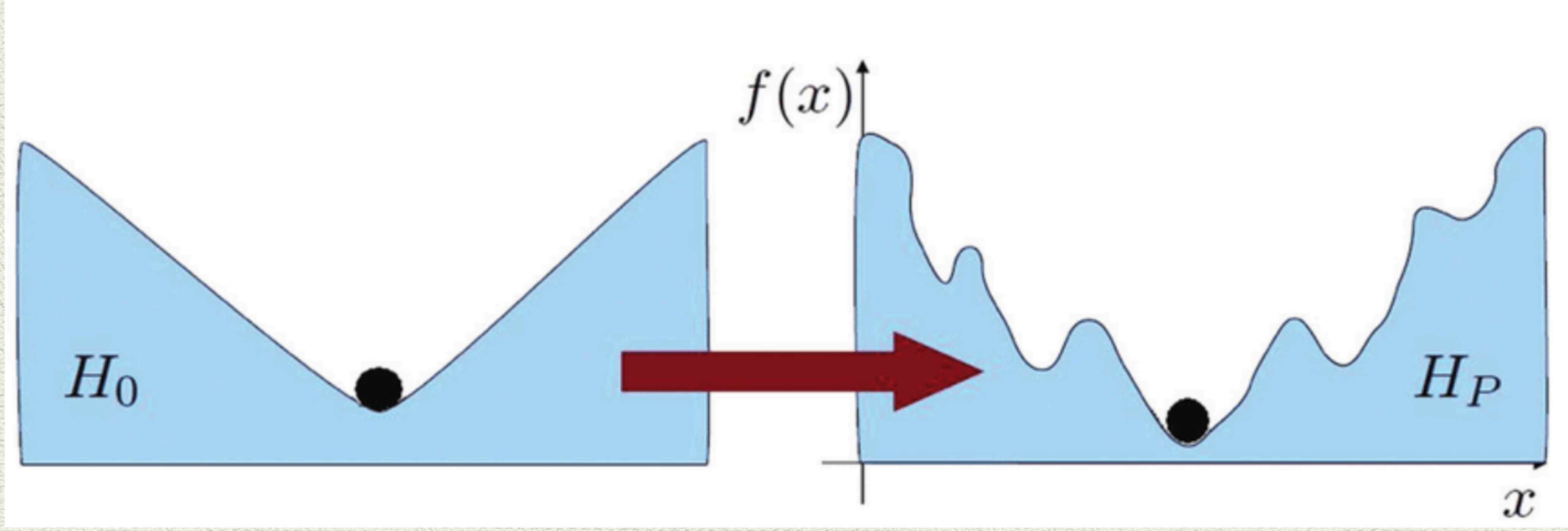
Quantum circuit model (Previous semseter)

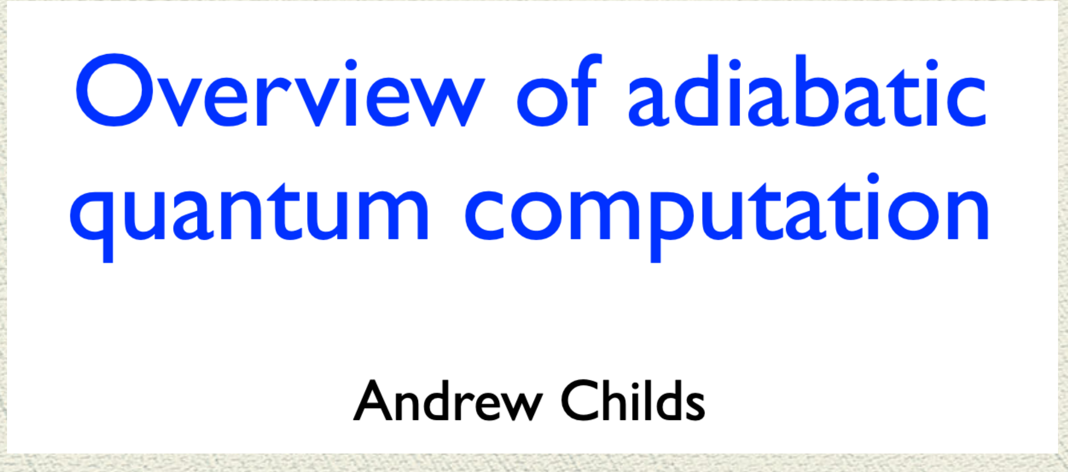
Adiabatic Quantum Computation

Measurement Based Quantum Computation(MBQC)

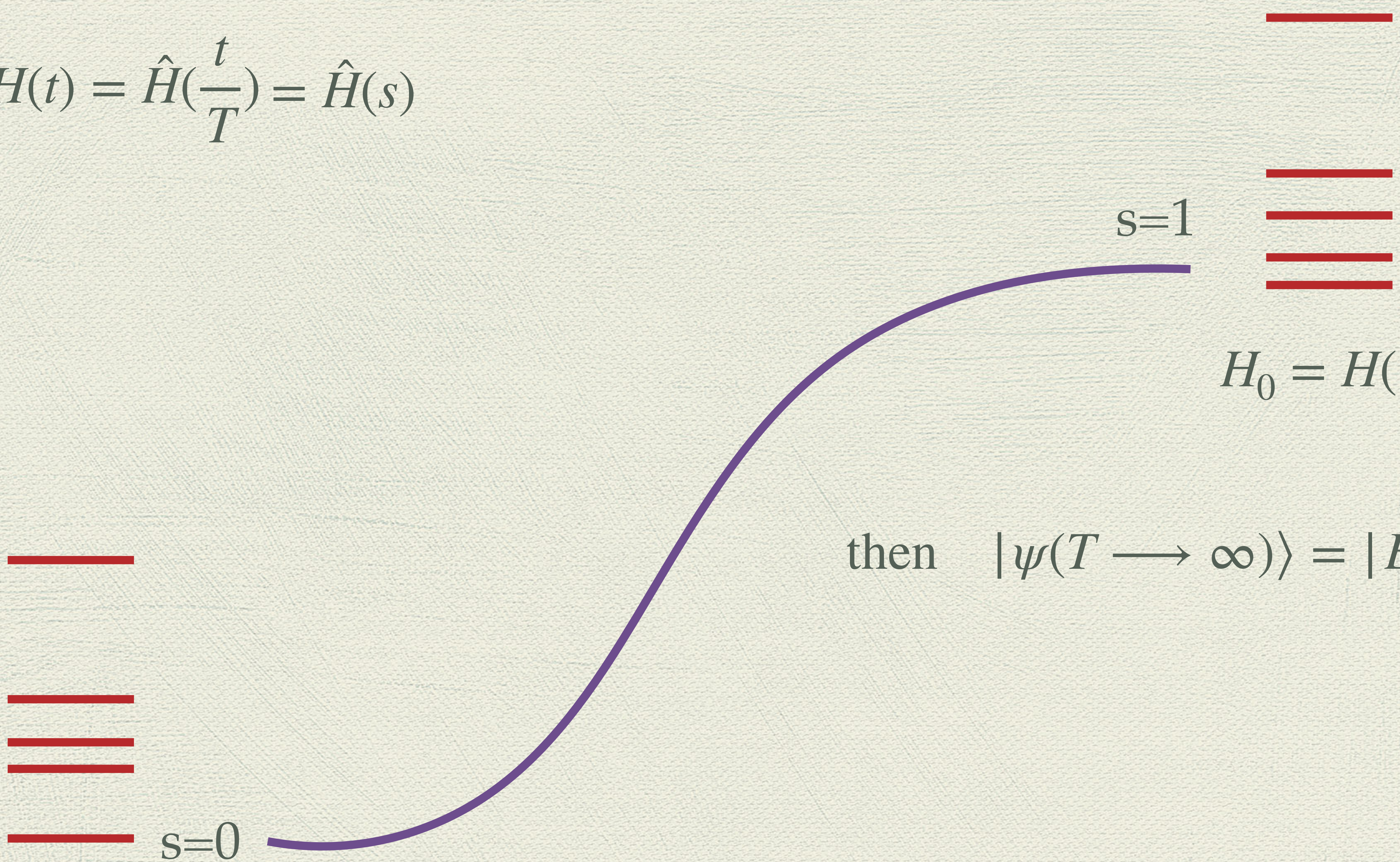
Topological Quantum Computation (This semester)

Adiabatic Quantum Computation





$$H(t) = \hat{H}(\frac{t}{T}) = \hat{H}(s)$$



$$H_0 = H(1)$$

then $\quad |\psi(T \longrightarrow \infty)\rangle = |E_0(1)\rangle$

$$H_0 = H(0)$$

if $\quad |\psi(0)\rangle = |E_0(0)\rangle$

3-SAT problem

$$f(s_1, s_2, s_3, s_4) = (s_1 \vee s_2 \vee s_3) \wedge (s_2 \vee \overline{s_3} \vee s_4)$$

$$g(s_1, s_2, s_3, s_4) = (s_1 \vee s_2 \vee s_3) \wedge (\overline{s_1} \vee \overline{s_2} \vee \overline{s_3})$$

$$h(s_1, s_2, s_3, s_4) = (s_1 \vee s_2 \vee s_3) \wedge (\overline{s_2} \vee \overline{s_3} \vee s_4) \wedge (s_2 \vee s_3 \vee \overline{s_4}) \wedge (s_2 \vee \overline{s} \vee \overline{s_4})$$

$$f(s_1, s_2, s_3, s_4) = (s_1 \vee s_2 \vee s_3) \wedge (s_2 \vee \overline{s_3} \vee s_4)$$

$$H = h_1 + h_2$$

$$h_1 = (1 - Z_1)(1 - Z_2)(1 - Z_3)$$

$$h_2 = (1 - Z_1)(1 + Z_3)(1 - Z_4)$$

$$H_1 = (1 - Z_1)(1 - Z_2)(1 - Z_3) + (1 - Z_2)(1 + Z_3)(1 - Z_4)$$

$$H_0 = -X_1 - X_2 - X_3 - X_4$$

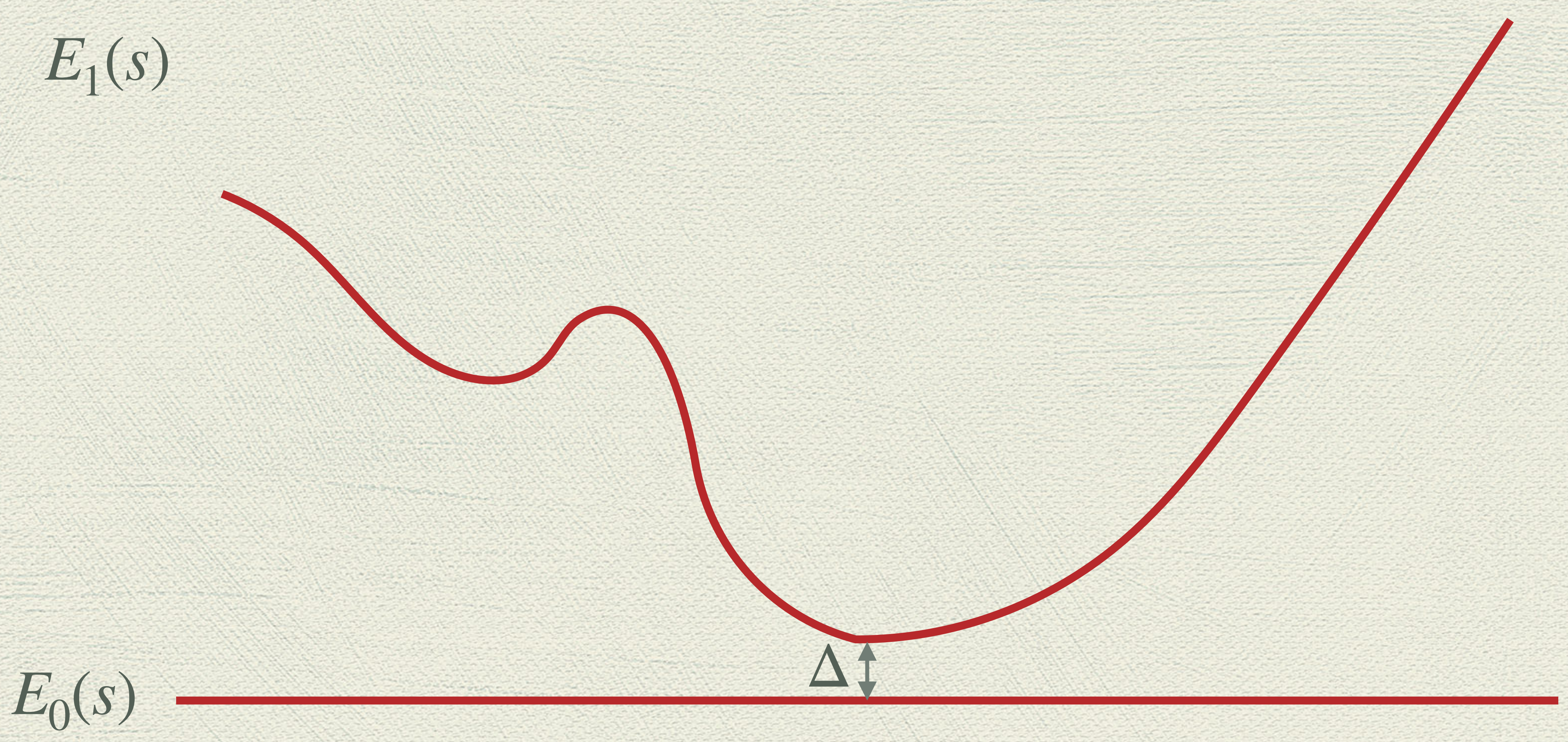
$$H_1 = (1 - Z_1)(1 - Z_2)(1 - Z_3) + (1 - Z_2)(1 + Z_3)(1 - Z_4)$$

$$|\psi_0\rangle = |+,+,+,+\rangle$$

$$|\psi_1\rangle = \{|1,1,1,1\rangle, \cdots |-1,1,1,-1\rangle, \cdots\}$$

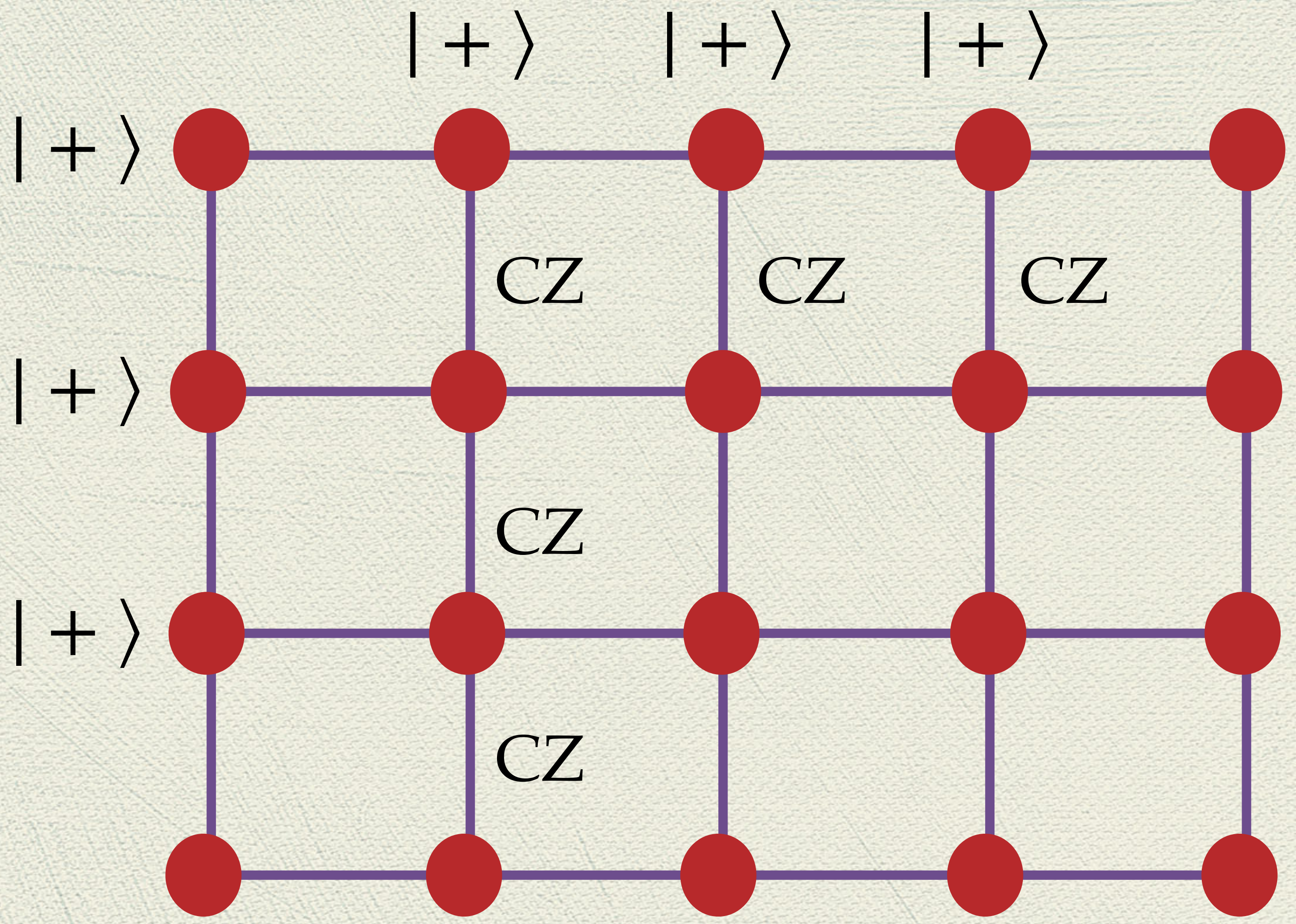
$$\tilde{H}(s) = (1 - s)H_0 + sH_1$$

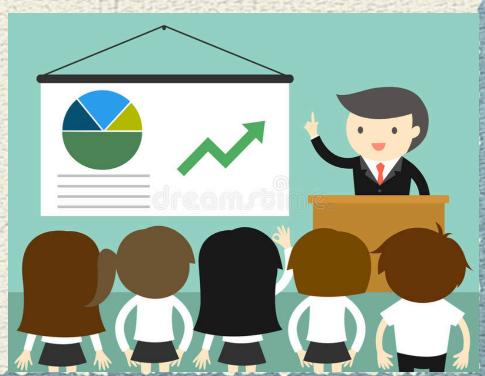
Energy Gap

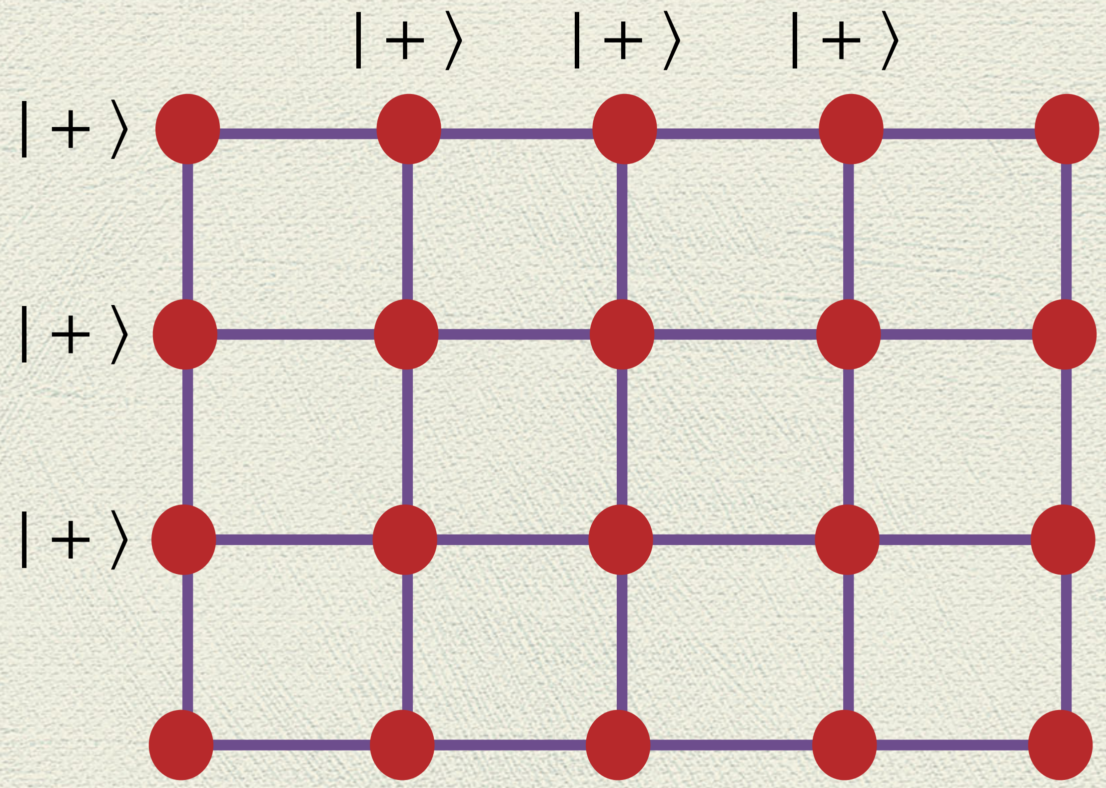


$$T \geq \frac{\Gamma^2}{\Delta^2} \qquad \Delta = \Delta(N)$$

Measurement Based Quantum Computation (MBQC)

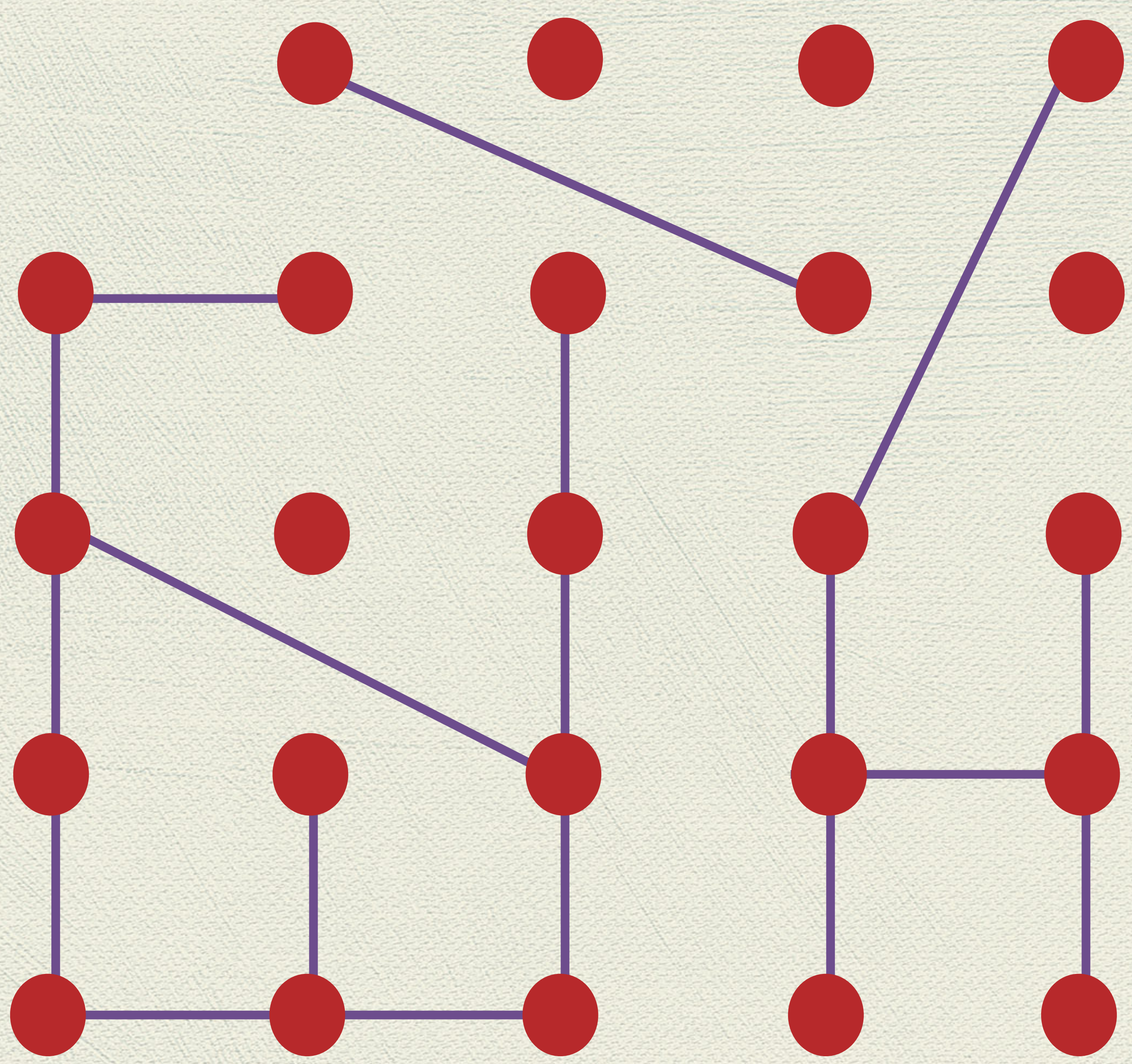


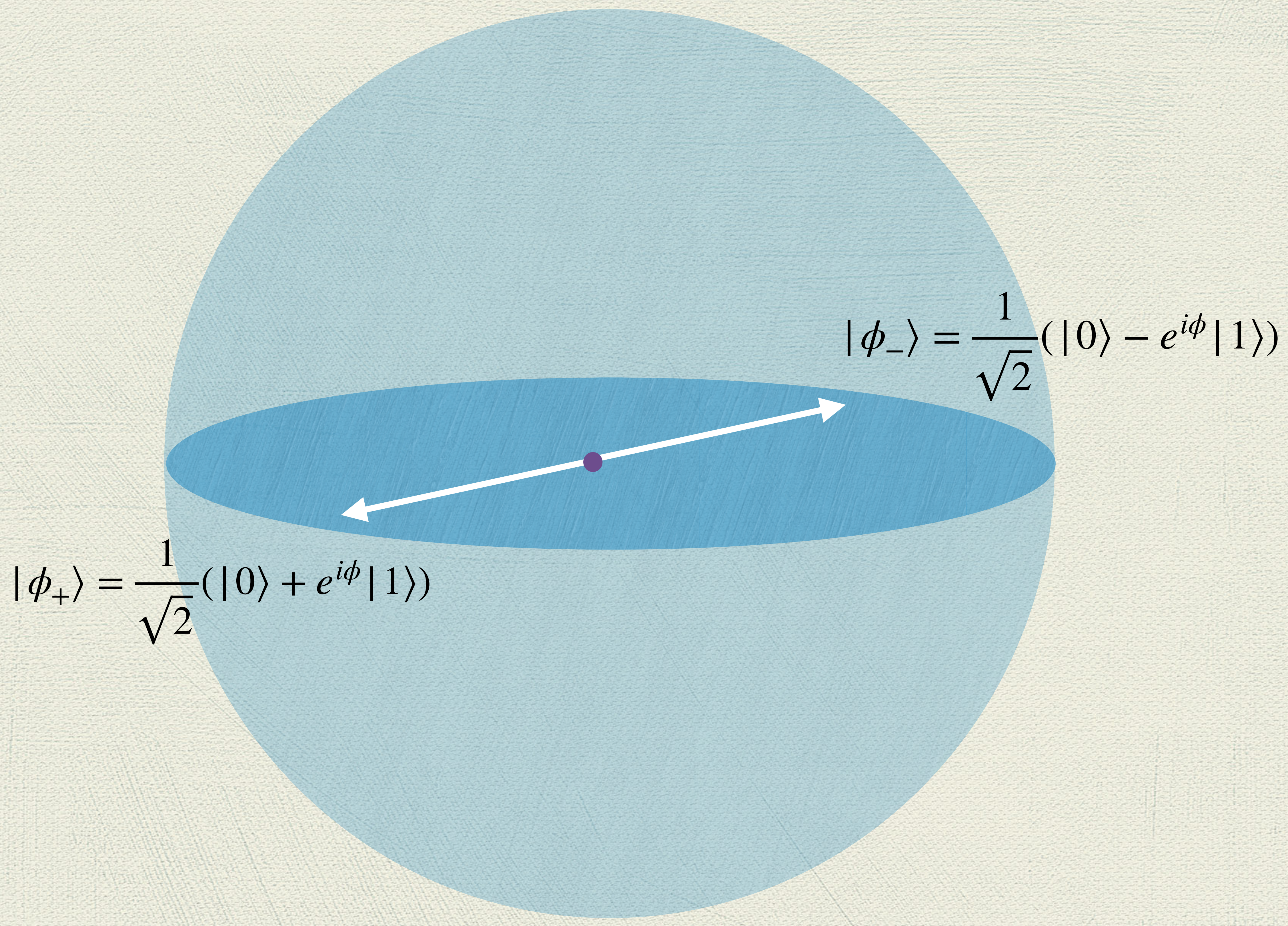




$$CZ = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

$$CZ|+,+\rangle = \frac{1}{2}\left(|0,0\rangle + |0,1\rangle + |1,0\rangle - |1,1\rangle\right)$$





$|\phi_-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - e^{i\phi}|1\rangle)$
$|\phi_+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$

$$|\phi_+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$$

$$|\phi_-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - e^{i\phi}|1\rangle)$$

$$|0\rangle = \frac{1}{\sqrt{2}}(|\phi_+\rangle + |\phi_-\rangle)$$

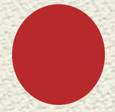
$$|1\rangle = \frac{1}{\sqrt{2}}e^{-i\phi}(|\phi_+\rangle - |\phi_-\rangle)$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |+\rangle$$

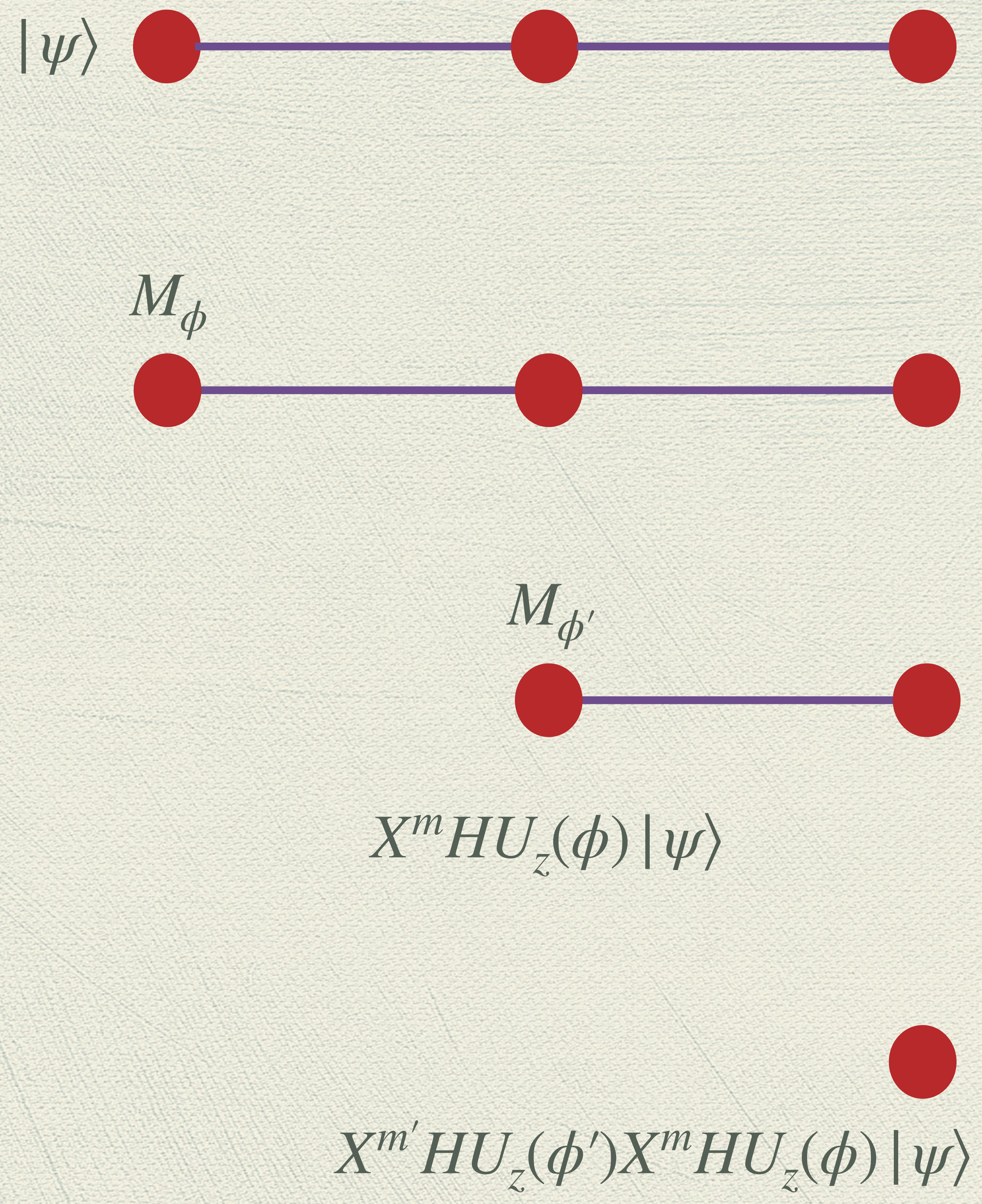
$$|\psi\rangle = \alpha|0,+\rangle + \beta|1,-\rangle$$

$$|\psi\rangle = \alpha|0,+\rangle + \beta|1,-\rangle$$

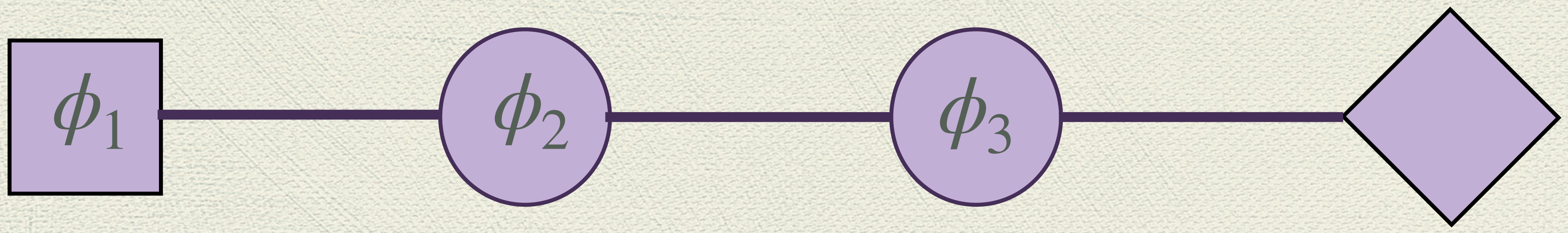
M_ϕ



$$X^m H U_z(\phi)|\psi\rangle$$


$|\psi\rangle$
M_ϕ
$M_{\phi'}$
$X^m H U_z(\phi)|\psi\rangle$
$X^{m'} H U_z(\phi') X^m H U_z(\phi)|\psi\rangle$

$X^m H U_z(\phi)$



$X^m H U_z(\phi_3) X^m H U_z(\phi_2) X^m H U_z(\phi_1)$

$$X^m H U_z(\phi_3) X^m H U_z(\phi_2) X^m H U_z(\phi_1)$$

$$=$$

$$X^{m_3} Z^{m_2} X^{m_1} H U_z((-1)^{m_2}\phi_3) U_x((-1)^{m_1}\phi_2) U_z(\phi_1)$$

$$\equiv$$

$$U_z(\gamma) U_x(\beta) U_z(\alpha)$$

One-way Quantum Computation

Dan Browne[a] and Hans Briegel[b]

[a]Departments of Materials and Physics, Oxford University, United Kingdom.
[b]Institute for Theoretical Physics, University of Innsbruck and Institute for Quantum Optics & Quantum Information (IQOQI) of the Austrian Academy of Sciences, Austria.

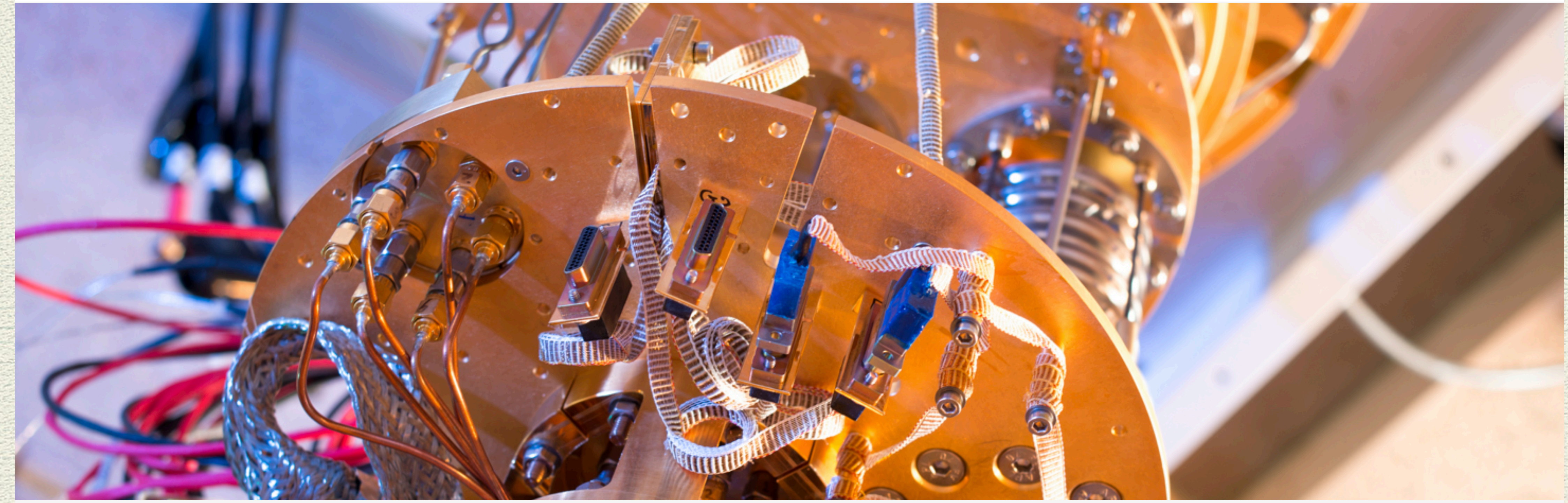
The one-way quantum computer – a non-network model of quantum computation

Robert Raussendorf, * Daniel E. Browne † and Hans J. Briegel ‡
Ludwig-Maximilians-Universität München

October 31, 2018

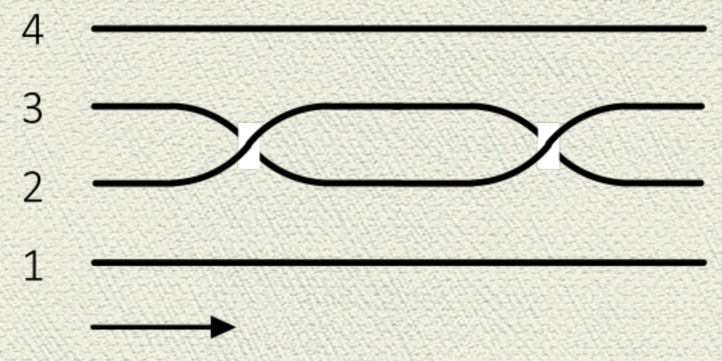
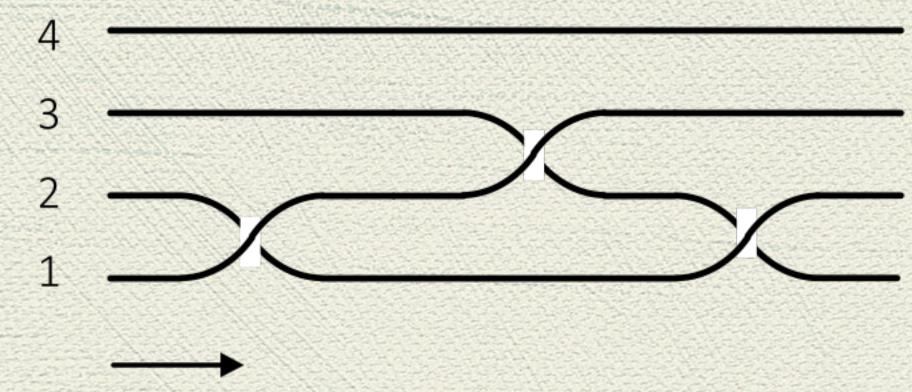
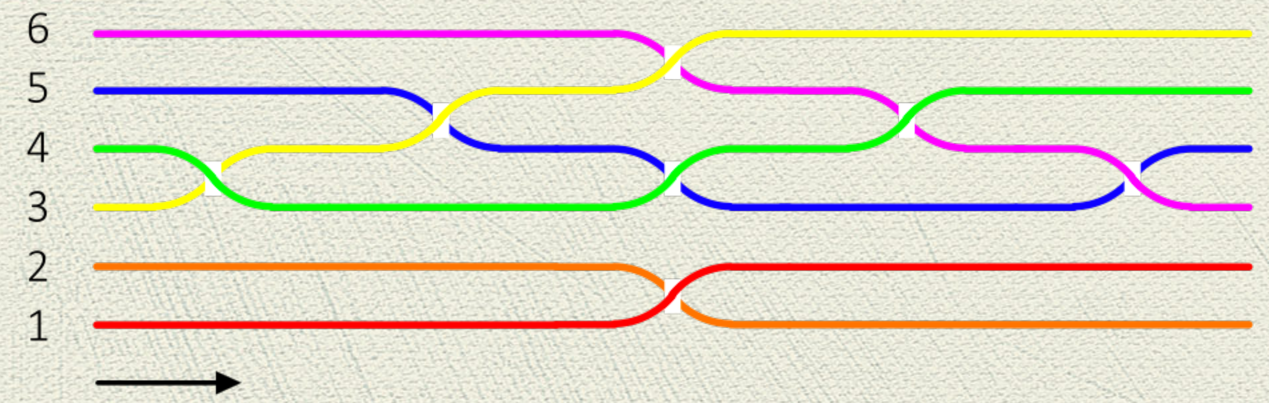
Topological Quantum Computation (this semester)

Microsoft | Stories Our Company News and Stories Press Tools All Microsoft Search Cart



With new Microsoft breakthroughs, general purpose quantum computing moves closer to reality

Topological Quantum Computation (this semester)

Braid diagram	Quantum gate
	 Pauli X quantum gate
	 Hadamard quantum gate
	 CNOT quantum gate

Gottesmann-Knill Theorem

$U \in$ Clifford Gates If $U\sigma_i U^\dagger = \sigma_j$

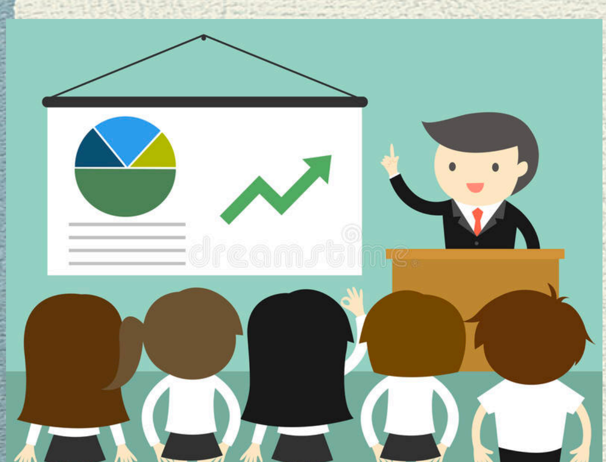
Clifford Gates are generated by **CNOT, H, S**

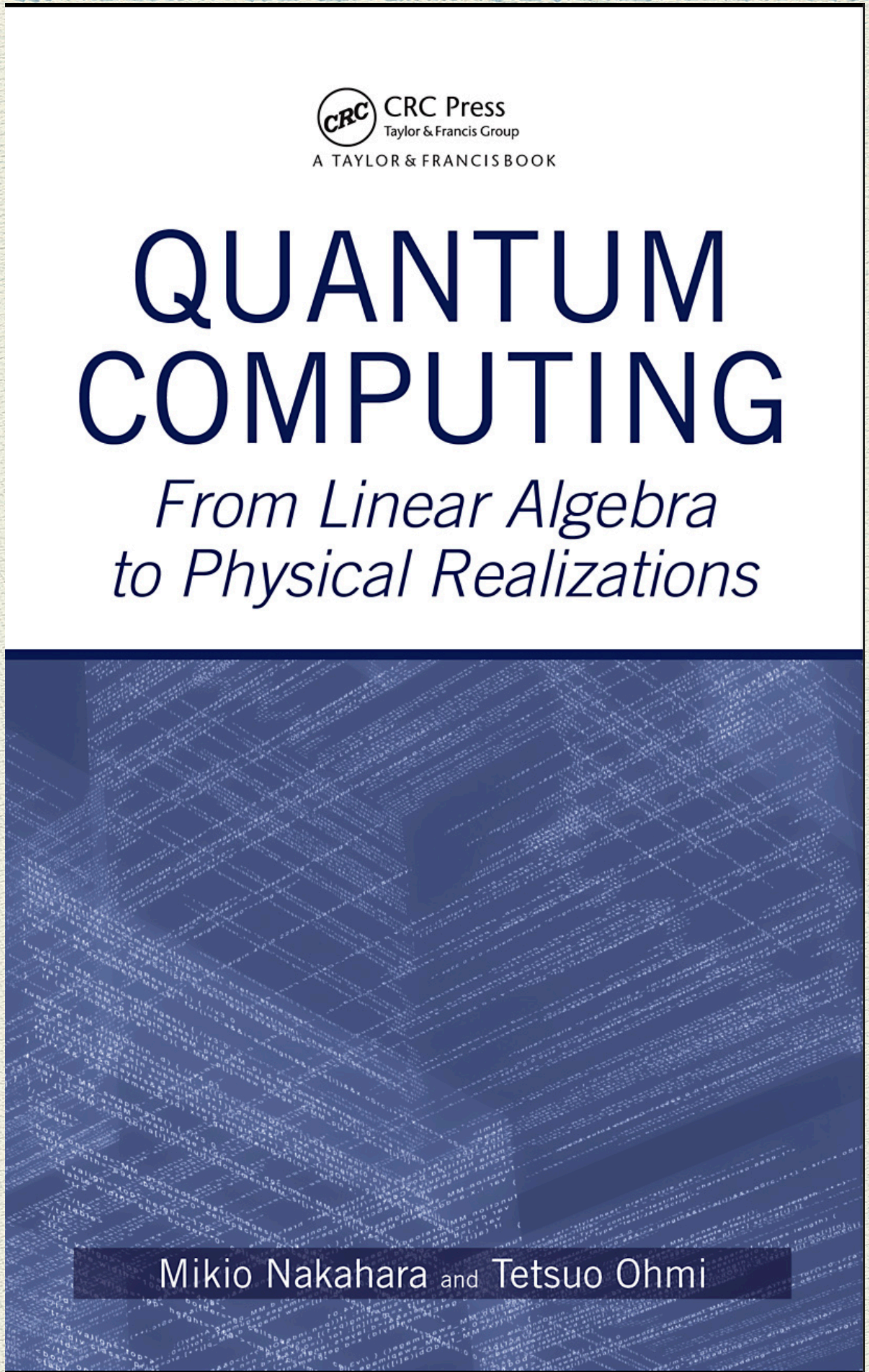
$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = \sqrt{Z}$$

Theorem: Any quantum circuit which is generated by Clifford Gates can be efficiently simulated by classical computers.

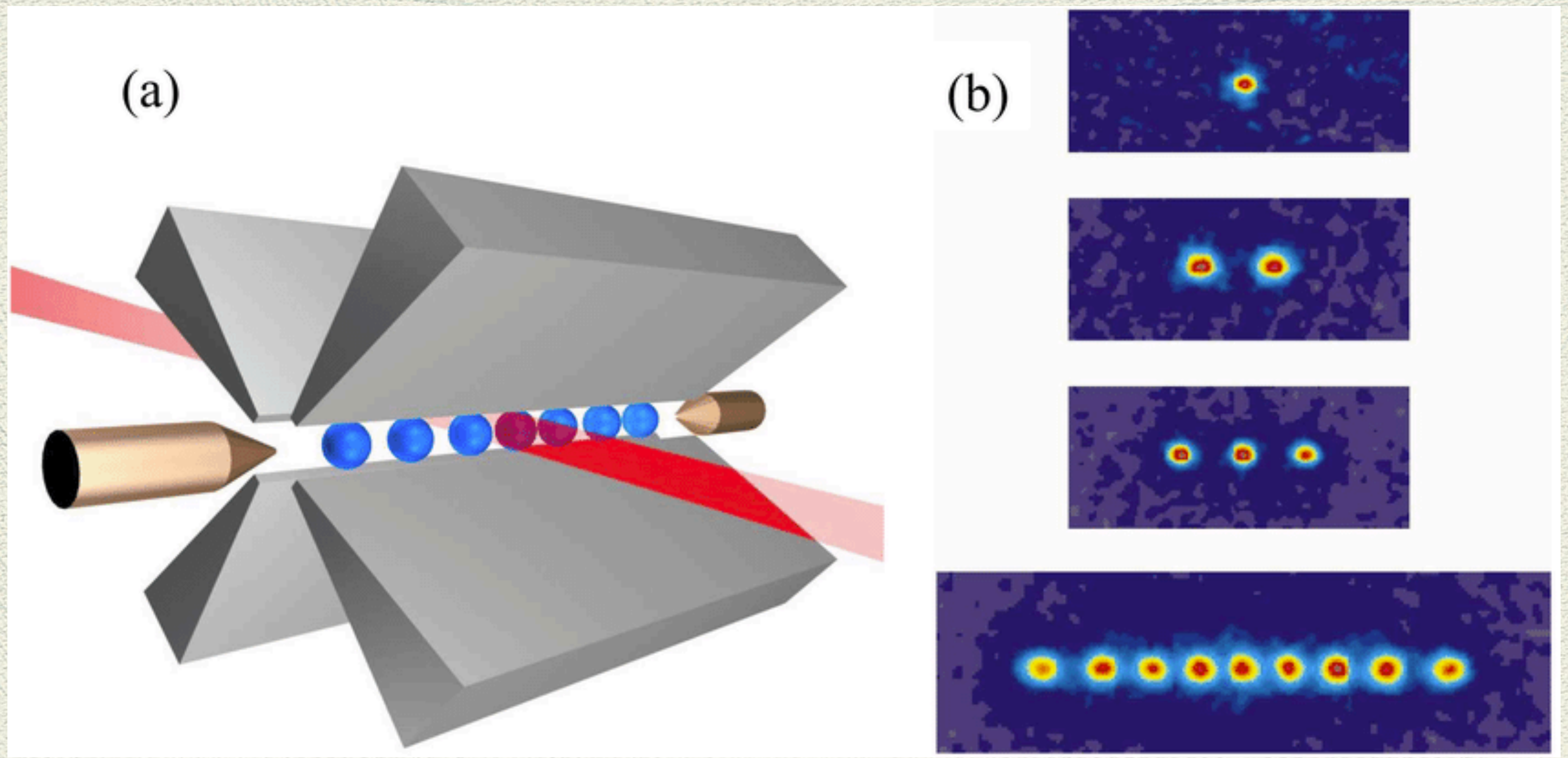
4- Quantum Hardware

- Ion Traps
- Superconducting qubits
- Cold Atoms





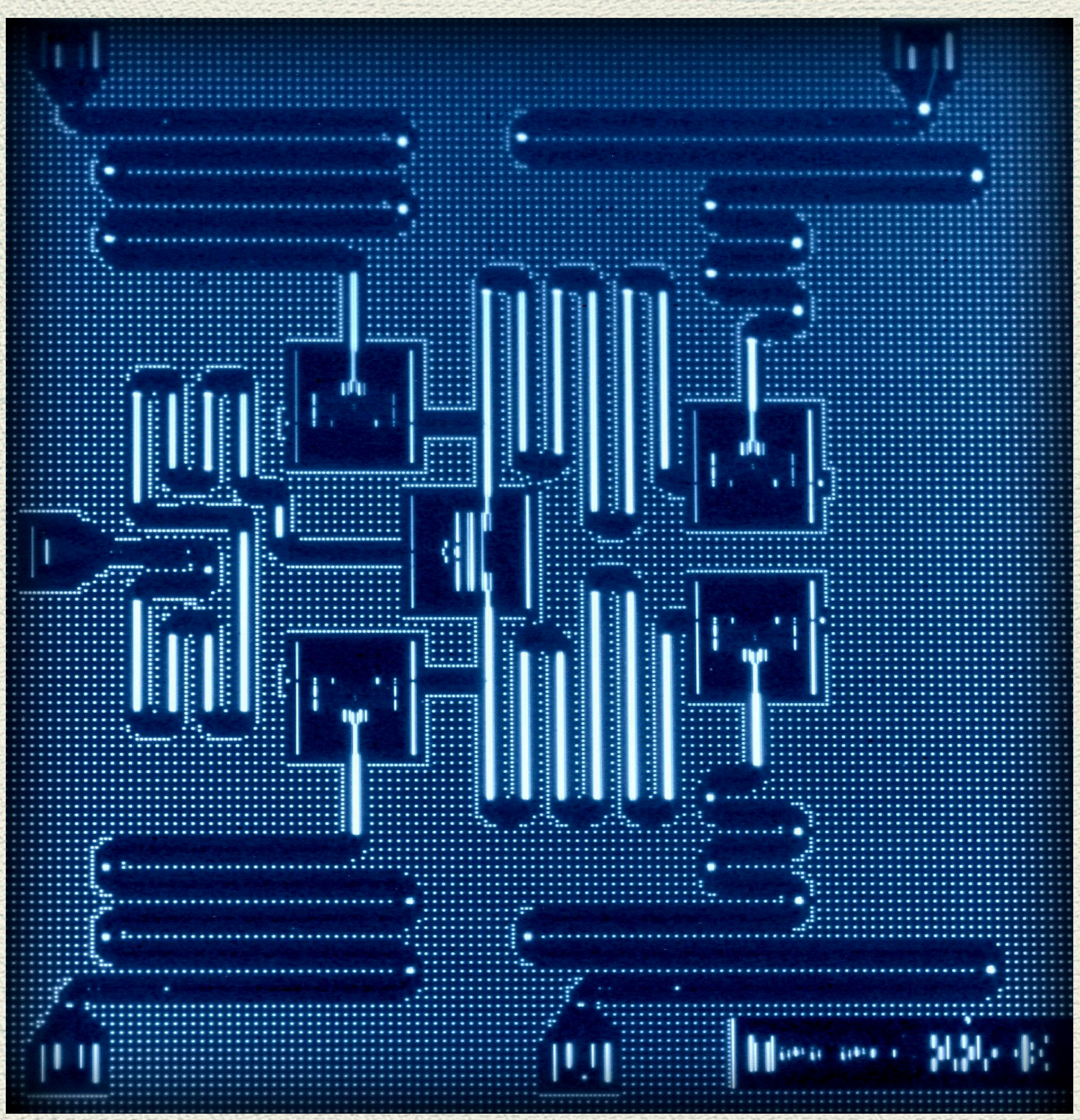
Ion traps (Chapter 13, Nakahara and Ohmi)



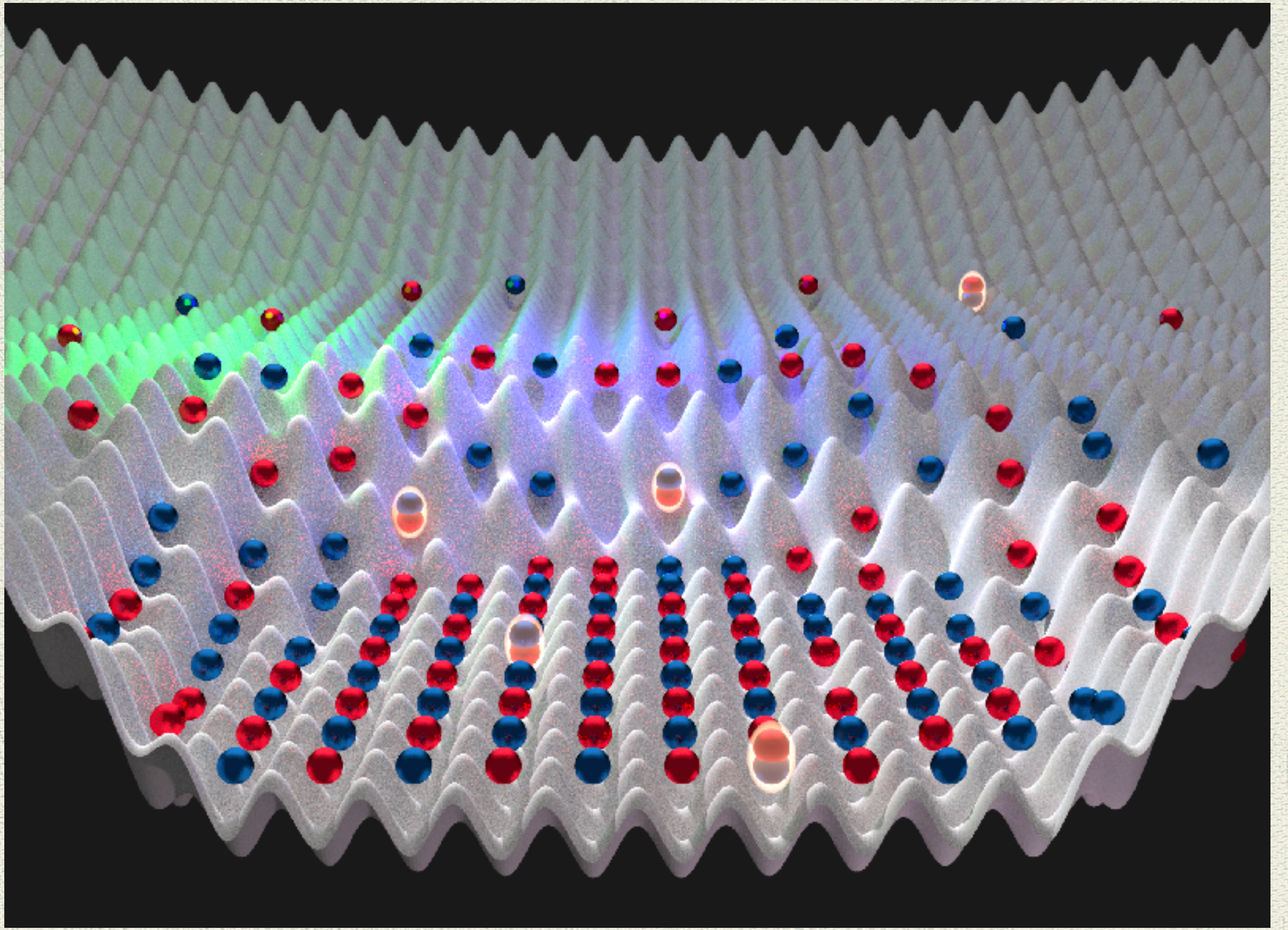
State preparation, Readout, single qubit gate

Two qubit operation

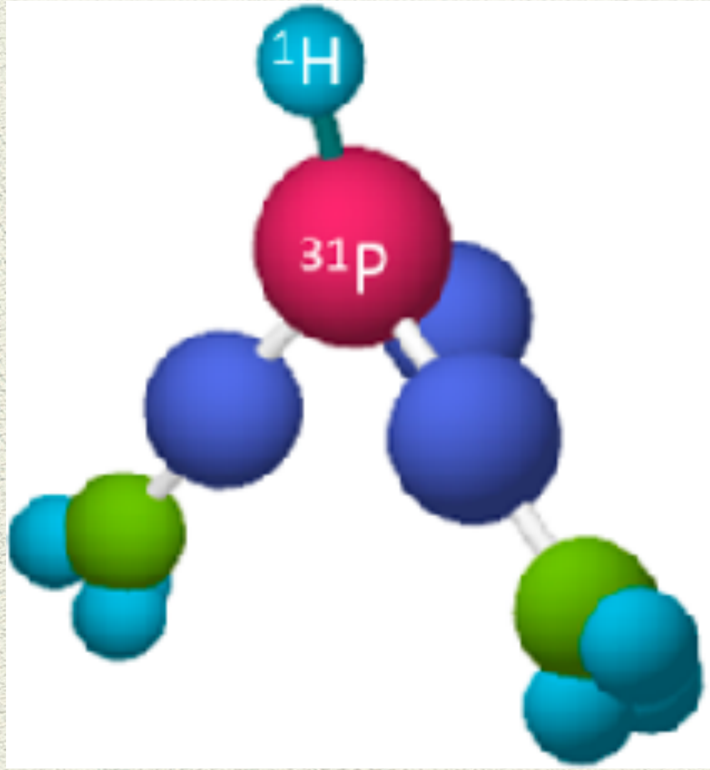
Superconducting qubits (Chapter 15 of Nakahara and Ohmi)



Cold atoms (Chapter 14 of Nakahara and Ohmi)



Nuclear Spins (Chapter 12 of Nakahara and Ohmi)

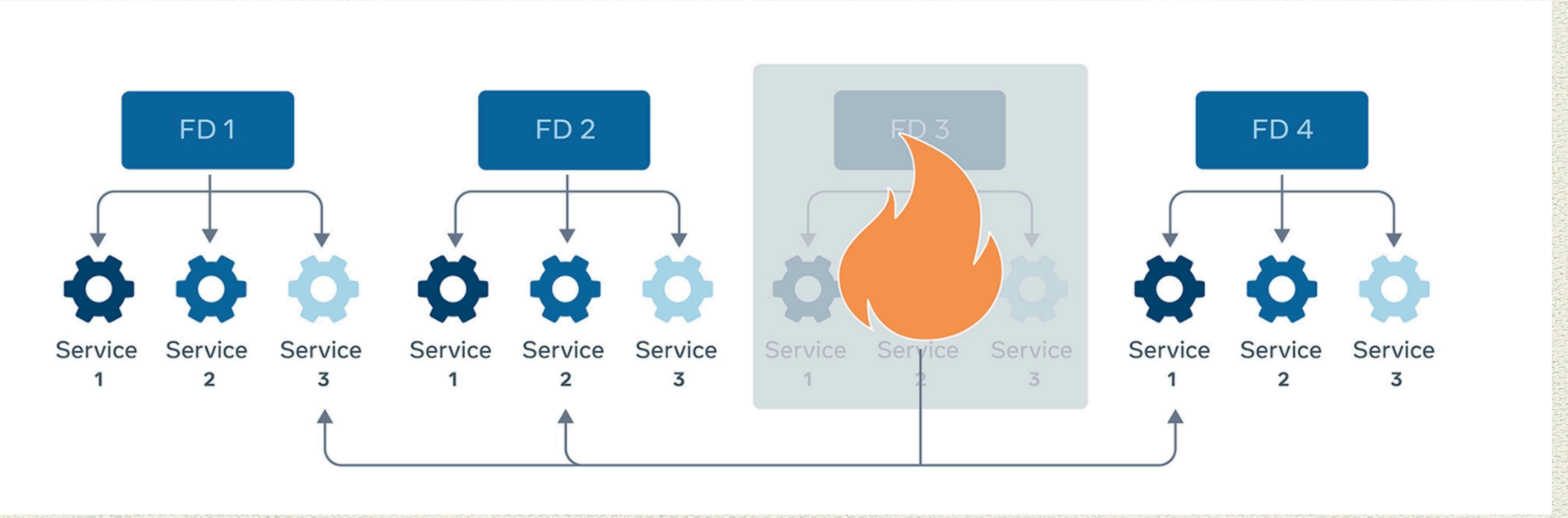


	^{1}H	^{31}P
^{1}H	0	697.4
^{31}P	697.4	0
T2 (s)	0.3	0.5
T1 (s)	4	7.2

5- Fault Tolerant Quantum Computing

A fault tolerant system (Power plant, Google, Dropbox,…)

Critical errors, Redundancy, Cost, Difficulty of diagnosis,…



Fault tolerant quantum computation

Concatenation

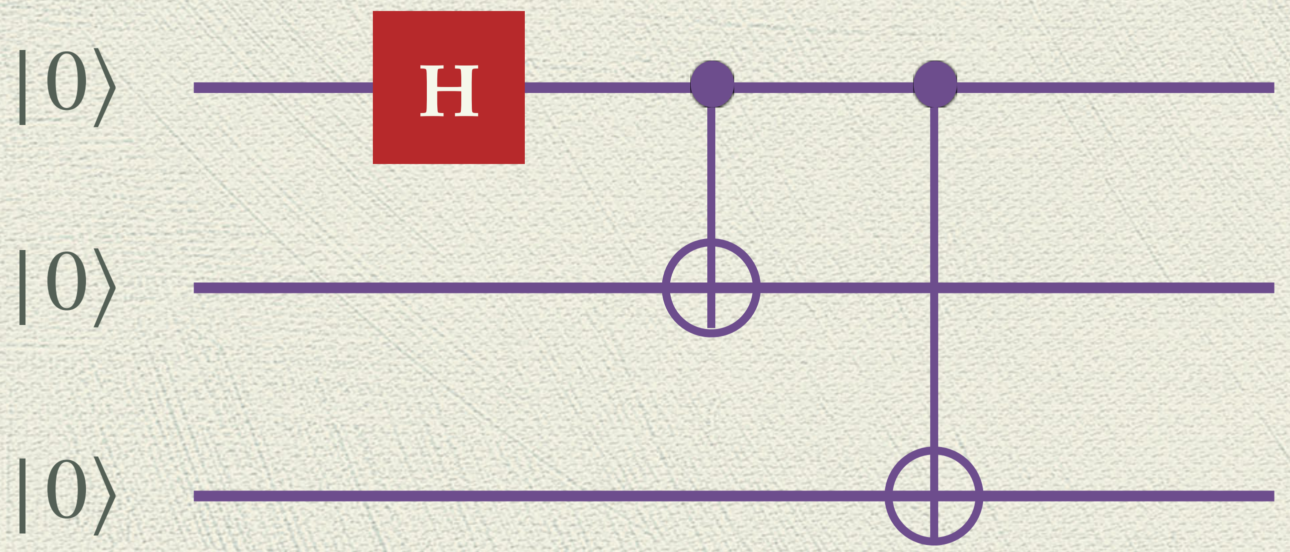
0	000	000 000 000
p	$3p^2$	$3(3p^2)^2 = 27p^4$

Error threshold

A simple code

$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

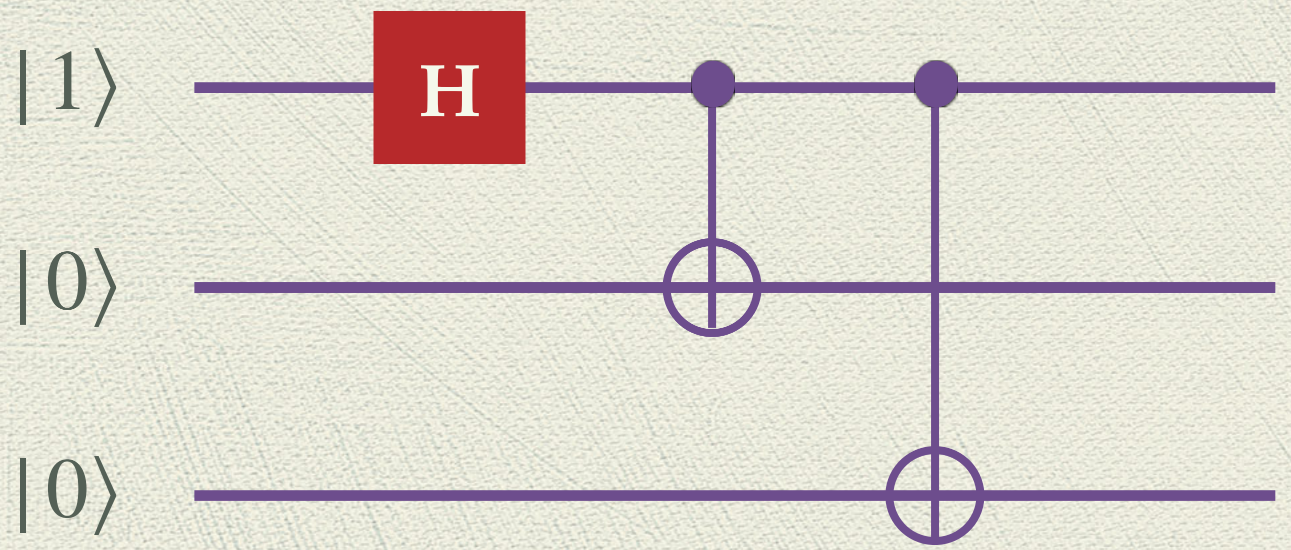
$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$$



$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

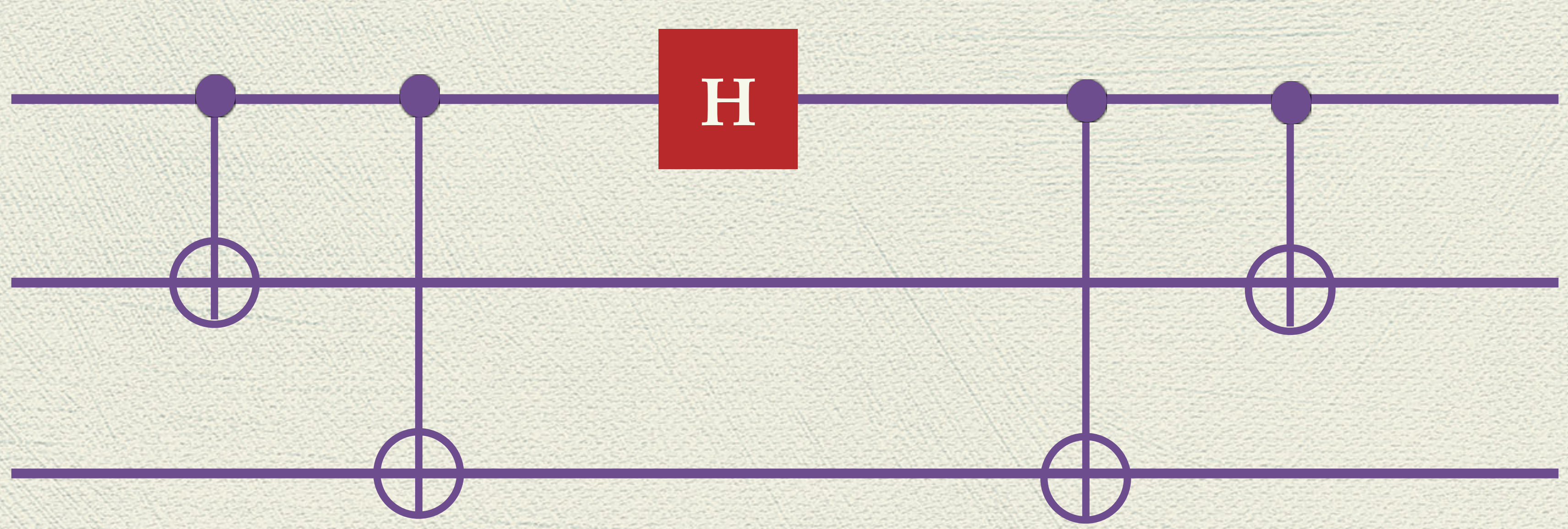
$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$$



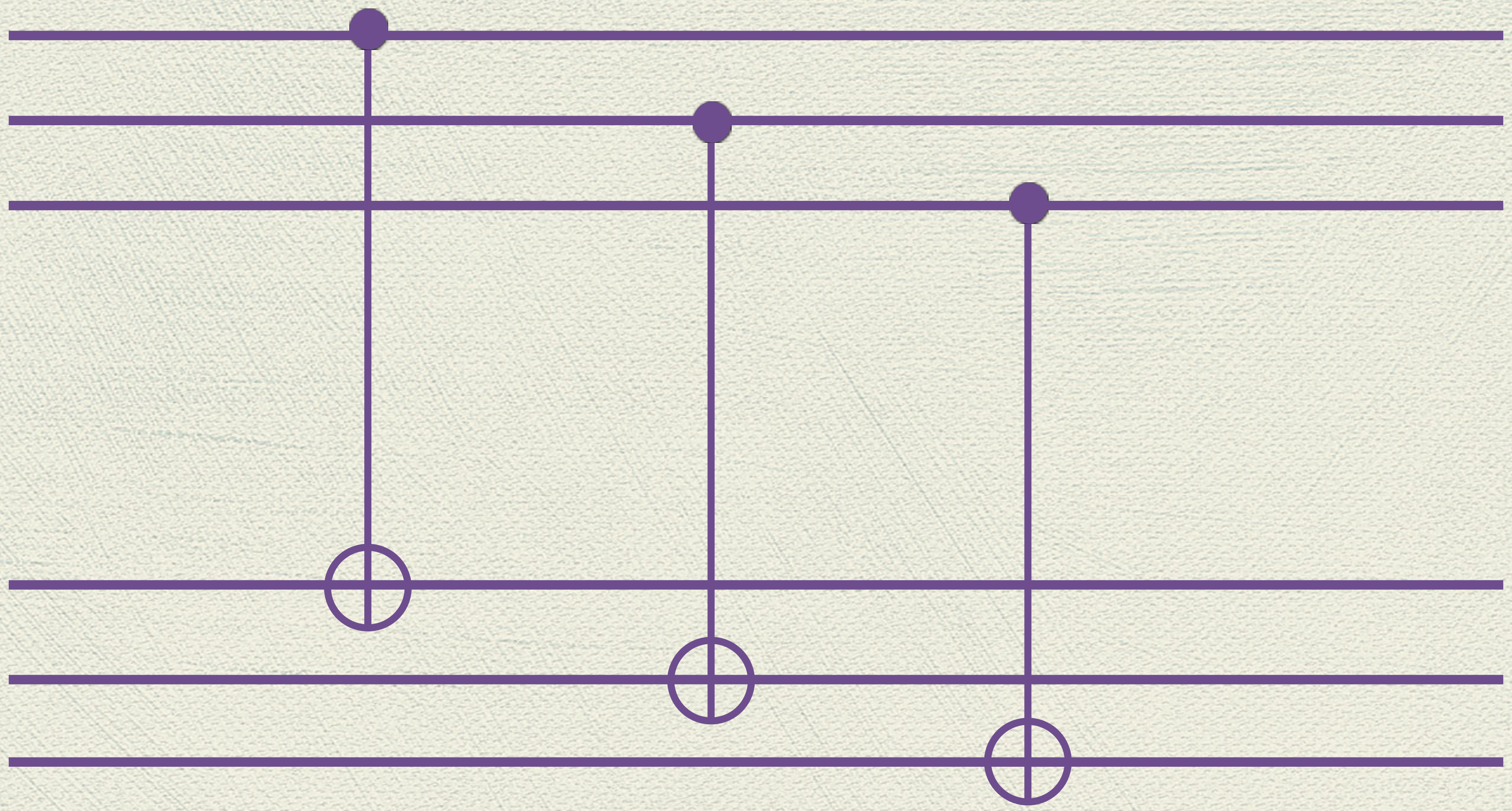
$$\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$$

Logical Gate



$H_{Logical}$

Transversal Gates



$CNOT_{Logical}$

Eastin-Knill Theorem

There is no quantum error correcting code for which there is a universal set of transversal gates.

100 Logical qubits → Surpassing classical computers,

Error threshold

100 Logical qubits = Millions of physical qubits

Fault-Tolerant Quantum Computation

Peter W. Shor
AT&T Research
Room 2D-149
600 Mountain Ave.
Murray Hill, NJ 07974, USA
shor@research.att.com

FAULT-TOLERANT QUANTUM COMPUTATION

JOHN PRESKILL
California Institute of Technology, Pasadena, CA 91125, USA

A Theory of Fault-Tolerant Quantum Computation

Daniel Gottesman*

California Institute of Technology, Pasadena, CA 91125

- Computational hardness of preparing ground states
- Entanglement dynamics in chaotic quantum systems
- Entanglement spreading